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STABILITY DERIVATIVES OF CONES AT SUPERSONIC SPEEDS

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SUMMARY

The aerodynamic stability derivatives due to pitching velocity and vertical acceleration are calculated by use of potential theory for circular cones traveling at supersonic speeds. The analysis is based on two theoretical techniques used successfully previously in application to the case of uniform axial and inclined flow. In the first, potential solutions for axial flow and crossflow are derived from the first-order wave equation but in application to calculations for the forces no approximations are made either to the tangency condition or to the isentropic pressure relation. The second method consists in combining the first-order crossflow potential with an axial-flow potential correct to second order. Closed-form solutions by both methods are found for a cone, and numerical results for the stability derivatives are presented as a function of Mach number for cones having semivertex angles of 10° and 20°.

In addition, expressions for the forces, moments, and stability derivatives of arbitrary bodies of revolution are obtained using Newtonian impact theory. Numerical results for cones compare well with those obtained from the combined first- and second-order potential theory at the highest Mach number for which the latter theory is applicable.

INTRODUCTION

The importance of the body as a lift-producing component of aircraft flying at supersonic speed has occasioned a great deal of theoretical work from which it is now possible to calculate the body's static aerodynamic properties with good accuracy (see, e.g., refs. 1 to 3 and attendant bibliographies). There is required, however, along with the static properties, theoretical information from which the dynamic behavior of bodies can be calculated, and in this field no work has been done that can be said to be applicable to nonslender bodies traveling at high supersonic Mach numbers. Slender-body theory, as is well known, fails to predict a dependence of the aerodynamic coefficients either on Mach number or on body shape (see, e.g., ref. 4). The work of Dorrance (ref. 5) based on the linear theory does indicate a Mach number dependence, but here too approximations made in the analysis effectively limit its application to bodies of vanishingly small thickness.

In this report, an attempt is made to overcome these limitations by adapting to the calculation of the body's rotary stability derivatives two theoretical methods derived by Van Dyke (ref. 1) that have proven successful in calculations for the static aerodynamic derivatives. It was shown by Van Dyke that the solution for normal force of a cone derived from the first-order potential equation may be greatly improved in accuracy (in comparison with the exact numerical results, ref. 2) if no approximations are made to the tangency condition or to the isentropic pressure relation. It was also shown that a further improvement could be realized by the use of a combination of first- and second-order potential solutions. The same ideas are used herein to calculate for a cone the stability derivatives due to pitching velocity, C_{N_q} and C_{m_q}, and due to vertical acceleration, $C_{N_{\dot{\alpha}}}$ and $C_{m_{\dot{\alpha}}}$. The results are believed to be of the same order of accuracy as those of reference 1 for the normal force due to angle of attack, and hence, for a cone of given thickness ratio, should apply to the same range of Mach numbers over which the normal force compares well with the Kopal results (ref. 2). Moreover, as in the angle-of-attack case, the cone solutions given herein are adaptable to the calculation of the stability derivatives of other more general body shapes by use of the techniques described in references 6 and 7.

In addition, expressions for the forces, moments, and stability derivatives of arbitrary bodies of revolution are obtained from Newtonian impact theory in order to furnish some information about the nature of these quantities at Mach numbers beyond the highest for which the results derived from potential theory are applicable.

NOTATION

a_o speed of sound in still air

C_N normal-force coefficient, $\frac{\text{normal force}}{q_o S}$

C_m pitching-moment coefficient, $\frac{\text{pitching moment}}{q_o S l}$

C_p pressure coefficient

C_X axial-force coefficient, $\frac{\text{axial force}}{q_o S}$

l body length

Mach number, $\frac{V}{a_o}$

q angular velocity (sketch (b))

q_o dynamic pressure, $\frac{1}{2}\rho_o V^2$

$R(x)$ radius of body in r,ω plane (sketch (a))

S body base area, $\pi R^2(l)$

t time

u,v,w free-stream velocity components relative to cylindrical coordinates fixed in body

x,r,ω cylindrical coordinates (sketch (a))

V velocity in axial direction of body

V_N component of free-stream velocity normal to body surface

α angle of attack

β $\sqrt{M^2 - 1}$

γ ratio of specific heats

θ slope of body meridian curve (sketch (a))

ρ_o density of still air

τ slope of cone surface, $\frac{R(l)}{l}$

Φ total potential

φ perturbation potential

φ_o first-order uniform axial flow potential

χ_o second-order uniform axial flow potential

Ω free-stream potential

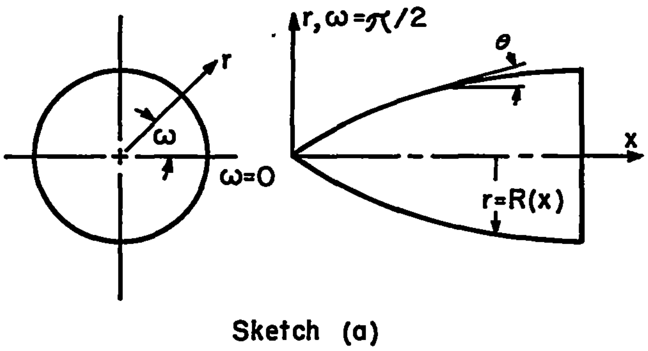
When α, $\dot{\alpha}$, and q are used as subscripts, a dimensionless derivative is indicated, and this derivative is evaluated as the independent variable (α, $\dot{\alpha}$, or q) approaches zero and all other variables are identically zero. Thus,

$$C_{m_\alpha} = \left(\frac{\partial C_m}{\partial \alpha}\right)_{\substack{\alpha \to 0 \\ \dot{\alpha}=q=0}}, \quad C_{m_{\dot{\alpha}}} = \left[\frac{\partial C_m}{\partial\left(\frac{\dot{\alpha} l}{V}\right)}\right]_{\substack{\dot{\alpha} \to 0 \\ \alpha=q=0}}, \quad C_{m_q} = \left[\frac{\partial C_m}{\partial\left(\frac{q l}{V}\right)}\right]_{\substack{q \to 0 \\ \alpha=\dot{\alpha}=0}}$$

ANALYSIS

Coordinate System and Definition of Free Stream

In the succeeding analysis we consider a pointed body of revolution flying at constant supersonic forward speed. Our purpose is to calculate the body's aerodynamic stability derivatives corresponding to the following motions: (1) sinking with uniform vertical velocity, (2) flying in a circular path with uniform angular velocity and at zero angle of attack, (3) sinking with uniform vertical acceleration. In order to define the motions conveniently, a cylindrical coordinate system is chosen that is fixed with respect to the body. As shown in sketch (a), the origin of the coordinate system is placed at the body apex. The positive branch of the x axis is coincident with the body's axis of revolution, and the coordinates r and ω are measured in a plane perpendicular to the x axis. With respect to this system of coordinates, the components of free-stream velocity u,v,w for the three motions are given below.



Sketch (a)

Sinking with uniform vertical velocity.- This case is of course equivalent to that of a stationary body situated in a uniform inclined stream and has already been treated extensively (refs. 1 to 3). It is included here again since the methods to be discussed subsequently for calculating the stability derivatives due to pitching velocity and vertical acceleration are in large part derived from the one used here. Let α be the angle of inclination of the stream with respect to the $\omega = 0$ plane; then the components of stream velocity in the axial, radial, and azimuthal directions, respectively, may be written,[1]

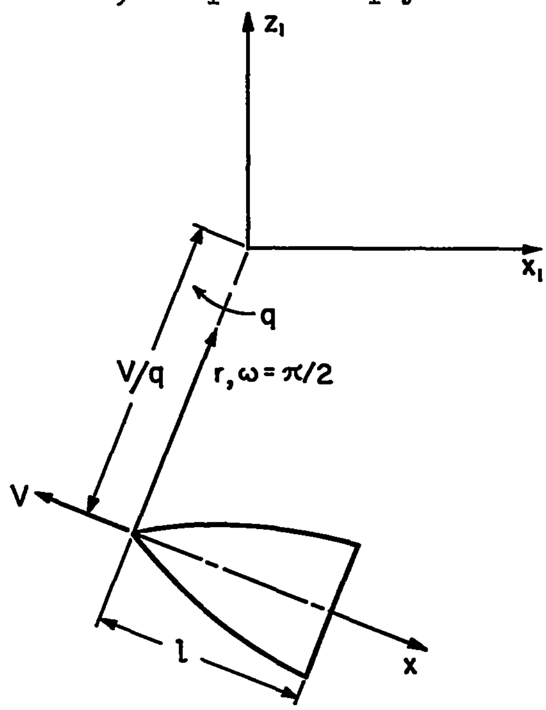
[1]Note that we have chosen to designate as V the axial component of velocity rather than the resultant flow velocity. It is necessary to do this in the subsequent cases, and we comply here for the sake of consistency. Note also that the radial and azimuthal velocities are measured in directions respectively normal and tangential to the body surface in the r,ω plane.

$$\left.\begin{aligned} u &= V \\ v &= V \tan \alpha \sin \omega \\ w &= V \tan \alpha \cos \omega \end{aligned}\right\} \quad (1)$$

Once the normal-force coefficient C_N and pitching-moment coefficient C_m corresponding to this motion have been calculated, the stability derivatives C_{N_α} and C_{m_α} are formed according to the definitions,

$$C_{N_\alpha} = \left(\frac{\partial C_N}{\partial \alpha}\right)_{\alpha \to 0}, \quad C_{m_\alpha} = \left(\frac{\partial C_m}{\partial \alpha}\right)_{\alpha \to 0}$$

Pitching with uniform angular velocity.- For this case, the body is considered to be flying in a circular path at zero angle of attack and with constant angular velocity q. The motion is, of course, that of the whirling-arm experiment, and, in terms of the latter case, we specify that the point of attachment of the body to the arm be at the body nose. With respect to a fixed system of coordinates, the pertinent physical dimensions are as shown in sketch (b). It will be assumed that the pitching rate q is small, so that $ql \ll V$. The radius of the flight path V/q, is then large compared with the body length. The body is assumed to have traveled far enough to have outrun its starting sound waves, but not so far as to have encountered its own wake, so that the flow may be said to be steady.



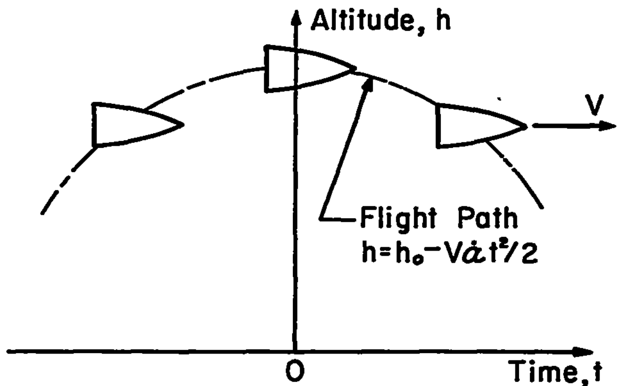
Sketch (b)

In the body system of coordinates, the components of stream velocity are,

$$\left.\begin{aligned} u &= V - qr \sin \omega \\ v &= qx \sin \omega \\ w &= qx \cos \omega \end{aligned}\right\} \quad (2)$$

The stability derivatives due to the motion are calculated according to the following definitions:

$$C_{N_q} = \left[\frac{\partial C_N}{\partial\left(\frac{ql}{V}\right)}\right]_{q \to 0} , \quad C_{m_q} = \left[\frac{\partial C_m}{\partial\left(\frac{ql}{V}\right)}\right]_{q \to 0}$$



Sketch (c)

Uniform vertical acceleration.- Since the body sinks (without pitching) at a linearly increasing rate, the flight path describes the parabolic arc shown in sketch (c). It is assumed that the motion has continued long enough so that transient effects have died out and, further, we choose to begin recording time at the point where the angle of attack is zero.

As viewed from the body-coordinate system, the components of free-stream velocity are,

$$\left.\begin{aligned} u &= V \\ v &= V\dot{\alpha}t \sin \omega \\ w &= V\dot{\alpha}t \cos \omega \end{aligned}\right\} \tag{3}$$

where $\dot{\alpha}$ is a constant. Note that although it has been assumed that transient effects associated with the start of the motion have disappeared, nevertheless, the flow in this case is unsteady, the crossflow velocities being linear functions of time.

The stability derivatives due to the motion are evaluated when the angle of attack is zero ($t = 0$) according to:

$$C_{N_{\dot{\alpha}}} = \left[\frac{\partial C_N}{\partial\left(\frac{\dot{\alpha}l}{V}\right)}\right]_{\dot{\alpha} \to 0} , \quad C_{m_{\dot{\alpha}}} = \left[\frac{\partial C_m}{\partial\left(\frac{\dot{\alpha}l}{V}\right)}\right]_{\dot{\alpha} \to 0}$$

Equations of Motion and Boundary Conditions

For the main part of this paper, the analysis will be based on the first-order steady and time-dependent potential equations for compressible flow. In body coordinates, the latter equation is,

$$\beta^2\Phi_{xx} - \Phi_{rr} - \frac{\Phi_{\omega\omega}}{r^2} - \frac{\Phi_r}{r} + \frac{2M}{a_o}\Phi_{xt} + \frac{1}{a_o^2}\Phi_{tt} = 0 \tag{4}$$

A perturbation potential φ is introduced such that

$$\Phi(x,r,\omega,t) = \Omega(x,r,\omega,t) + \varphi(x,r,\omega,t) \tag{5}$$

where Ω is the free-stream potential. Direct substitution gives for the first-order time-dependent perturbation equation,

$$\beta^2\varphi_{xx} - \varphi_{rr} - \frac{\varphi_{\omega\omega}}{r^2} - \frac{\varphi_r}{r} + \frac{2M}{a_o}\varphi_{xt} + \frac{1}{a_o^2}\varphi_{tt} = 0 \tag{6}$$

The steady-state counterpart of equation (6) is obtained by eliminating time derivatives, so that,

$$\beta^2\varphi_{xx} - \varphi_{rr} - \frac{\varphi_{\omega\omega}}{r^2} - \frac{\varphi_r}{r} = 0 \tag{7}$$

Equation (7) will be used to determine solutions for the stability derivatives due to steady sinking and steady pitching, whereas the stability derivatives due to vertical acceleration are derived from equation (6). In either case, however, the boundary conditions have the same form, namely, that,

(1) Velocity perturbations vanish on the Mach cone emanating from the body nose:

$$\varphi\left(x, \frac{x}{\beta}, \omega, t\right) = 0 \tag{8}$$

(2) The flow velocity normal to the body surface is identically zero:

$$\frac{dR}{dx} = \frac{\varphi_r(x,R,\omega,t) + \Omega_r(x,R,\omega,t)}{\varphi_x(x,R,\omega,t) + \Omega_x(x,R,\omega,t)} \tag{9}$$

The velocities Ω_r and Ω_x are, of course, the free-stream radial and axial velocity components, respectively, so that more conveniently,

$$\frac{dR}{dx} = \frac{\varphi_r(x,R,\omega,t) + v(x,R,\omega,t)}{\varphi_x(x,R,\omega,t) + u(x,R,\omega,t)} \tag{10}$$

where, for the three motions to be considered, u and v are given by equations (1) to (3). Note also that we designate the independent radial coordinate by r, whereas in evaluation of conditions at the body surface, the fact that r is a function of x is indicated by the use of R (i.e., $r = R = R(x)$).

First-Order Solutions for the Potential

We consider next the task of finding solutions for the perturbation potential satisfying equations (6) or (7) and compatible with the boundary conditions, equations (8) and (9), corresponding to each of the three motions.

Sinking with uniform vertical velocity.- As mentioned previously, the method of solution in this case is well known; however, since the methods to be used for the two subsequent cases derive from the one used here, a brief account of the essential steps is given below.

The steady perturbation potential is broken into two parts: A potential φ_o independent of ω and hence corresponding to a uniform axial flow, and a crossflow potential φ_1. The total perturbation potential is then the sum of φ_o and φ_1. For the axial-flow potential φ_o, the equation of motion is,

$$\varphi_{o_{rr}} + \frac{\varphi_{o_r}}{r} - \beta^2\varphi_{o_{xx}} = 0 \tag{11}$$

with the boundary conditions (from eqs. (1), (8), and (10))

$$\varphi_o\left(x, \frac{x}{\beta}\right) = 0 \tag{12}$$

$$\frac{dR}{dx} = \frac{\varphi_{o_r}(x,R)}{\varphi_{o_x}(x,R) + V} \tag{13}$$

A solution to equation (11) that automatically satisfies (12) is (ref. 6),

$$\varphi_o(x,r) = \int_{\cosh^{-1}\frac{x}{\beta r}}^{0} f(x - \beta r \cosh u)\,du \tag{14}$$

where $f(x - \beta r \cosh u)$ represents the distribution of sources along the x axis, and is to be chosen such that equation (14) satisfies the boundary condition (13).

For the crossflow potential φ_1, the equation of motion is,

$$\varphi_{1_{rr}} + \frac{\varphi_{1_r}}{r} + \frac{\varphi_{1_{\omega\omega}}}{r^2} - \beta^2 \varphi_{1_{xx}} = 0 \tag{15}$$

with the boundary conditions (eqs. (1), (8), and (10)),

$$\varphi_1\left(x, \frac{x}{\beta}, \omega\right) = 0 \tag{16}$$

$$\frac{dR}{dx} = \frac{\varphi_{1_r}(x,R,\omega) + V \tan \alpha \sin \omega}{\varphi_{1_x}(x,R,\omega)} \tag{17}$$

A solution to equation (15) that automatically satisfies (16) is (ref. 6),

$$\varphi_1(x,r,\omega) = -\beta \sin \omega \int_{\cosh^{-1} \frac{x}{\beta r}}^{0} m(x - \beta r \cosh u) \cosh u \, du \tag{18}$$

where $m(x - \beta r \cosh u)$ represents the distribution of doublets along the x axis, and is to be chosen such that equation (18) satisfies the boundary condition (17).

<u>Pitching with uniform angular velocity</u>.- The procedure in this case parallels the above development. Again, the perturbation potential φ is broken into a uniform axial flow potential φ_o and a potential φ_2, each having the same equation of motion as equations (11) and (15), respectively. Note, however, that unlike the previous example, the calculation for the potential φ_2 must take into account the nonuniform <u>axial</u> component of stream velocity, $-qr \sin \omega$ (eq. (2)). The boundary conditions on φ_o are, from equations (2), (8), and (10),

$$\varphi_o\left(x, \frac{x}{\beta}\right) = 0 \tag{19}$$

$$\frac{dR}{dx} = \frac{\varphi_{o_r}(x,R)}{\varphi_{o_x}(x,R) + V} \tag{20}$$

while the boundary conditions on φ_2 are,

$$\varphi_2\left(x, \frac{x}{\beta}, \omega\right) = 0 \tag{21}$$

$$\frac{dR}{dx} = \frac{\varphi_{2_r}(x,R,\omega) + qx \sin \omega}{\varphi_{2_x}(x,R,\omega) - qR \sin \omega} \tag{22}$$

The solutions for the potentials φ_o and φ_2 of course have the same form as equations (14) and (18), respectively, where $f(x - \beta r \cosh u)$ and $m(x - \beta r \cosh u)$ are to be chosen such that equations (20) and (22) are satisfied.

Sinking with uniform vertical acceleration.- As mentioned previously, this is a problem in unsteady flow, since the crossflow velocities v and w are functions of time. It is still possible, however, to consider the potential φ in two parts; an axial component φ_o that is independent of both t and ω, and hence is again governed by equation (11), and a crossflow component φ_3, governed in this case by equation (6), with $\varphi = \varphi_3$.

For the axial-flow potential, we proceed exactly as in the two previous cases. The boundary conditions to be satisfied are, from equations (3), (8), and (10),

$$\varphi_o\left(x, \frac{x}{\beta}\right) = 0 \tag{23}$$

$$\frac{dR}{dx} = \frac{\varphi_{o_r}(x,R)}{V + \varphi_{o_x}(x,R)} \tag{24}$$

The solution for φ_o is given by equation (14) where $f(x - \beta r \cosh u)$ is chosen such that equation (24) is satisfied.

For the unsteady crossflow potential φ_3, we adapt a concept used previously in wing theory by Ribner and Malvestuto (ref. 8) and originated by C. S. Gardner. It is easily verified by substitution into equation (6) that the following relation satisfies the unsteady potential equation no matter what the constant K may be:

$$\frac{\varphi_3}{\dot{\alpha}} = K\psi(x,r,\omega) + \left(t - \frac{M^2 x}{\beta^2 V}\right)\chi(x,r,\omega) \tag{25}$$

where

ψ steady-state potential for unit pitching velocity about body apex, $\frac{\varphi_2}{q}$

χ steady-state potential for unit angle of attack, $\frac{\varphi_1}{\tan \alpha}$

Note that the potentials involved in equation (25) are steady-state potentials and, further, are just the potentials φ_1 and φ_2 developed in the two previous sections. Having found a solution for the potential, we then must satisfy the boundary conditions which, from equations (3), (8), and (10), are

$$\varphi_3\left(x, \frac{x}{\beta}, \omega, t\right) = 0 \tag{26}$$

$$\frac{dR}{dx} = \frac{\varphi_{3_r}(x,R,\omega,t) + V\dot{\alpha}t \sin \omega}{\varphi_{3_x}(x,R,\omega,t)} \tag{27}$$

Equation (26) is satisfied immediately, since the potentials ψ and χ individually satisfy it. The tangency condition, equation (27), is then readily satisfied by proper choice of the constant K.

Pressure, Force, and Moment Coefficients

It will have been noticed in the previous section that we have specified the exact form of the tangency condition for each of the motions considered. The same will be done for the pressure relations, to be given below. Following Van Dyke (ref. 1), the view taken on this point is simply that approximations to the tangency and pressure relations, while justifiable mathematically on an order basis, serve to impair unnecessarily the accuracy of the solution in comparison with known numerical results. While this is known to be the case only for the uniformly sinking motion, the formulation of the problem for the other motions has introduced no further approximations, and hence it is entirely reasonable to suppose that the same order of accuracy will be realized in these cases as well if the pressure and tangency relations are not approximated.

The pressure coefficient must be defined separately for each of the three motions; this has been done in the appendix, and the results are repeated here for convenience.

Sinking with uniform vertical velocity.-

$$C_{p_1} = \frac{2}{\gamma M^2}\left[\left(1 - \frac{\gamma - 1}{2} M^2 \Lambda_1\right)^{\frac{\gamma}{\gamma-1}} - 1\right] \tag{28}$$

where,

$$\Lambda_1 = \frac{2\varphi_x}{V} + 2 \tan \alpha\left(\frac{\varphi_\omega}{rV} \cos \omega + \frac{\varphi_r}{V} \sin \omega\right) + \frac{\varphi_x^2 + \varphi_r^2 + (\varphi_\omega/r)^2}{V^2}$$

and r is taken as $R(x)$ when evaluating C_p at the body surface.

Pitching with uniform angular velocity.-

$$C_{p_2} = \frac{2}{\gamma M^2}\left[\left(1 - \frac{\gamma - 1}{2} M^2 \Lambda_2\right)^{\frac{\gamma}{\gamma-1}} - 1\right] \tag{29}$$

where

$$\Lambda_2 = \frac{2\varphi_x}{V} - \frac{2\varphi_x}{V^2} qr \sin \omega + \frac{2qx}{V}\left(\frac{\varphi_\omega}{rV} \cos \omega + \frac{\varphi_r}{V} \sin \omega\right) + \left[\frac{\varphi_x^2 + \varphi_r^2 + (\varphi_\omega/r)^2}{V^2}\right]$$

Sinking with uniform vertical acceleration.-

$$C_{p_3} = \frac{2}{\gamma M^2}\left[\left(1 - \frac{\gamma - 1}{2} M^2 \Lambda_3\right)^{\frac{\gamma}{\gamma-1}} - 1\right] \tag{30}$$

where

$$\Lambda_3 = \frac{2\varphi_x}{V} + \frac{2\varphi_t}{V} + 2\dot{\alpha}t\left(\frac{\varphi_\omega}{rV} \cos \omega + \frac{\varphi_r}{V} \sin \omega\right) + \left[\frac{\varphi_x^2 + \varphi_r^2 + (\varphi_\omega/r)^2}{V^2}\right]$$

Normal-force and pitching-moment coefficients.- Once having the pressure coefficient, one can determine the normal-force and pitching-moment coefficients from the following relations:

$$C_N = -\frac{2}{S}\int_0^l R(x)dx \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} C_p(x,R,\omega)\sin \omega \, d\omega \tag{31}$$

$$C_m = \frac{2}{Sl}\int_0^l \left[xR(x) + \tan\,\theta R^2(x)\right]dx \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} C_p(x,R,\omega)\sin\,\omega\;d\omega \tag{32}$$

where the pitching moment is referred to an axis through the body nose.

Stability Derivatives for a Cone - First-Order Theory

The foregoing equations for potential and pressure coefficient are, in principle, applicable to the determination of forces, moments, and stability derivatives of arbitrarily shaped bodies of revolution. In order to complete the analysis, it is necessary to find distributions of sources and doublets $f(x - \beta r\,\cosh\,u)$ and $m(x - \beta r\,\cosh\,u)$ that are compatible with the boundary conditions corresponding to the specified body shape. Unfortunately, analytical expressions for these quantities have been found only for the cone. However, it has been shown in the axial- and inclined-flow cases (refs. 6 and 7) how, by the use of summation techniques, the cone solution can be used to find solutions for other more general body shapes to any desired accuracy. The same techniques are readily adaptable to the other motions considered herein. Therefore, as a necessary beginning toward obtaining the stability derivatives of more general body shapes, the calculations for the cone are carried out below for each of the three motions under consideration.

Sinking with uniform vertical velocity.- Consider first the potential for axial flow φ_o, given by equation (14). For the case of a cone, whose surface is given by $R = \tau x$, an appropriate distribution of sources is simply (ref. 6),

$$f(x - \beta r\,\cosh\,u) = A_o(x - \beta r\,\cosh\,u) \tag{33}$$

where A_o is a constant, to be determined from the boundary condition, equation (13). Integrating equation (14), and substituting the appropriate derivatives in equation (13), we get for A_o,

$$A_o = \frac{V\tau^2}{\tau^2\cosh^{-1}\frac{1}{\beta\tau} + \sqrt{1 - \beta^2\tau^2}} \tag{34}$$

whereupon the axial-flow potential is,

$$\varphi_o(x,r) = A_o\left(\sqrt{x^2 - \beta^2 r^2} - x\,\cosh^{-1}\frac{x}{\beta r}\right) \tag{35}$$

The computation for the crossflow potential φ_1 proceeds in the same way. An appropriate distribution of doublets is found to be (ref. 6)

$$m(x - \beta r \cosh u) = A_1(x - \beta r \cosh u) \tag{36}$$

Substituting in equation (18), integrating, and solving for A_1 in equation (17),

$$A_1 = \frac{2V\tau^2 \tan \alpha}{(2\tau^2 + 1)\sqrt{1 - \beta^2\tau^2} + \beta^2\tau^2 \cosh^{-1} \frac{1}{\beta\tau}} \tag{37}$$

and

$$\varphi_1(x,r,\omega) = A_1 \frac{\beta}{2} \sin \omega \left(\frac{x}{\beta r}\sqrt{x^2 - \beta^2 r^2} - \beta r \cosh^{-1} \frac{x}{\beta r}\right) \tag{38}$$

The perturbation potential φ is then given by the sum of equations (35) and (38).

Having obtained the potential, one can now compute the pressure coefficient by equation (28). However, it is more convenient to proceed directly to the calculation of the stability derivatives C_{N_α} and C_{m_α}, by means of the expressions,

$$\left.\begin{aligned}
C_{N_\alpha} &= \left(\frac{\partial C_N}{\partial \alpha}\right)_{\alpha \to 0} = -\frac{2}{S}\int_0^l R(x)dx \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\frac{\partial C_{p_1}}{\partial \alpha}\right)_{\alpha \to 0} \sin \omega \, d\omega \\
C_{m_\alpha} &= \left(\frac{\partial C_m}{\partial \alpha}\right)_{\alpha \to 0} = \frac{2}{Sl}\int_0^l [xR(x) + \tan \theta \, R^2(x)]dx \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\frac{\partial C_{p_1}}{\partial \alpha}\right)_{\alpha \to 0} \sin \omega \, d\omega
\end{aligned}\right\} \tag{39}$$

where

$$\left(\frac{\partial C_{p_1}}{\partial \alpha}\right)_{\alpha \to 0} = -\left[1 - \frac{\gamma - 1}{2} M^2 (\Lambda_1)_{\alpha \to 0}\right]^{\frac{1}{\gamma - 1}} \left(\frac{\partial \Lambda_1}{\partial \alpha}\right)_{\alpha \to 0}$$

The quantity $\left(\frac{\partial C_{p_1}}{\partial \alpha}\right)_{\alpha \to 0}$ is computed by substituting the appropriate derivatives of φ into equation (28) and differentiating with respect to α as indicated. Substituting the results into equation (39) and integrating gives (letting $\gamma = 1.4$),

$$\left.\begin{aligned} C_{N_\alpha} &= \left(\frac{2\beta^2 \Gamma_1}{\Delta + \beta^2}\right)\left(\frac{1 + \tau^2}{1 + \Delta + 2\tau^2}\right) \\ C_{m_\alpha} &\doteq -\frac{2}{3}(1 + \tau^2) C_{N_\alpha} \end{aligned}\right\} \tag{40}$$

where

$$\Delta = \frac{\beta^2 \tau^2 \cosh^{-1} \frac{1}{\beta\tau}}{\sqrt{1 - \beta^2 \tau^2}}$$

$$\Gamma_1 = \left\{1 + 0.2M^2 \left[1 - (1 + \tau^2)\left(\frac{\beta^2}{\Delta + \beta^2}\right)^2\right]\right\}^{2.5}$$

The above results reduce to the slender-body-theory result as the thickness parameter τ approaches zero:

$$\left.\begin{aligned} C_{N_\alpha} &= 2 \\ C_{m_\alpha} &= -\frac{4}{3} \end{aligned}\right\} \tag{41}$$

<u>Pitching with uniform angular velocity</u>.- Again, consider first the uniform axial flow potential φ_0. There is nothing new to compute, however, since the equation of motion and boundary conditions are the same for all three cases. The potential φ_0 is, therefore, given by equation (35).

For the potential φ_2 (eq. (18)) a distribution of doublets that satisfies the boundary condition, equation (22), is,

$$m(x - \beta r \cosh u) = B_2(x - \beta r \cosh u)^2 \tag{42}$$

Substituting equation (42) in (18), integrating, and solving for B_2 in equation (22), we get,

$$\varphi_2(x,r,\omega) = B_2 \sin \omega \left\{\left[\frac{\beta^3 r^2}{3}\left(\frac{x^2}{\beta^2 r^2} + 2\right)\sqrt{\left(\frac{x}{\beta r}\right)^2 - 1}\right] - \beta^2 rx \cosh^{-1}\frac{x}{\beta r}\right\} \tag{43}$$

where

$$B_2 = \frac{-q\tau^2(1 + \tau^2)}{(\tau^2 - 1)\left(\beta^2\tau^2 \cosh^{-1}\frac{1}{\beta\tau} - \sqrt{1 - \beta^2\tau^2}\right) - \frac{4}{3}(1 - \beta^2\tau^2)^{3/2}}$$

As before, the total perturbation potential φ is the sum of φ_o (eq. (35)) and φ_2 (eq. (43)). Also, in the same way, the stability derivatives C_{N_q} and C_{m_q} are computed according to,

$$\left.\begin{aligned}
C_{N_q} &= \left(\frac{\partial C_N}{\partial \frac{ql}{V}}\right)_{q \to o} = -\frac{2}{S}\int_o^l R(x)dx \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left(\frac{\partial C_{p_2}}{\partial \frac{ql}{V}}\right)_{q \to o} \sin \omega \, d\omega \\
C_{m_q} &= \left(\frac{\partial C_m}{\partial \frac{ql}{V}}\right)_{q \to o} = \frac{2}{Sl}\int_o^l [xR(x) + \tan\theta\, R^2(x)]dx \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left(\frac{\partial C_{p_2}}{\partial \frac{ql}{V}}\right)_{q \to o} \sin \omega \, d\omega
\end{aligned}\right\} \tag{44}$$

where C_{p_2} is given by equation (29). There results,

$$\left.\begin{aligned} C_{N_q} &= \frac{2}{3}\Gamma_1\left\{1 + 2\left(\frac{\beta^2}{\Delta + \beta^2}\right)(1 + \tau^2)\left[\frac{3(1-\Delta) - 2(1-\beta^2\tau^2)}{4(1-\beta^2\tau^2) - 3(1-\tau^2)(1-\Delta)}\right]\right\} \\ C_{m_q} &= -\frac{3}{4}(1 + \tau^2)C_{N_q} \end{aligned}\right\} \tag{45}$$

where Δ and Γ_1 have been defined in equations (40).

As in the previous case, these results reduce to the slender-body-theory result as $\tau \rightarrow 0$;

$$\left.\begin{aligned} C_{N_q} &= 2 \\ C_{m_q} &= -\frac{3}{2} \end{aligned}\right\} \tag{46}$$

<u>Sinking with uniform vertical acceleration</u>.- The axial-flow potential φ_0 is again given by equation (35). For the time-dependent cross-flow potential φ_3, use is made of equation (25). Inserting the values for φ_1 and φ_2 obtained in the two previous cases into equation (25) gives,

$$\begin{aligned} \frac{\varphi_3}{\dot{\alpha}} = K\left(\frac{B_2}{q}\right)\sin\omega\left\{\sqrt{\left(\frac{x}{\beta r}\right)^2 - 1}\left[\frac{\beta^3 r^2}{3}\left(\frac{x^2}{\beta^2 r^2} + 2\right)\right] - \beta^2 rx\cosh^{-1}\frac{x}{\beta r}\right\} + \\ \left(t - \frac{M^2 x}{V\beta^2}\right)\left(\frac{A_1}{\tan\alpha}\right)\left(\frac{\sin\omega}{2}\right)\left[\beta x\sqrt{\left(\frac{x}{\beta r}\right)^2 - 1} - \beta^2 r\cosh^{-1}\frac{x}{\beta r}\right] \end{aligned} \tag{47}$$

where B_2/q and $A_1/\tan\alpha$ have been defined in equations (43) and (37). The constant K is then determined by substituting the appropriate derivatives of φ_3 into the boundary condition, equation (27). There results,

$$K = \frac{M^2}{\beta^2(1 + \tau^2)}\left[\frac{(3\tau^2+1)\sqrt{1-\beta^2\tau^2} + (1 - \tau^2)\beta^2\tau^2\cosh^{-1}\frac{1}{\beta\tau}}{(2\tau^2+1)\sqrt{1-\beta^2\tau^2} + \beta^2\tau^2\cosh^{-1}\frac{1}{\beta\tau}}\right] \tag{48}$$

As before, the sum of φ_o (eq. (35)) and φ_3 (eq. (47)) is then the total perturbation potential φ due to uniform vertical acceleration.

The stability derivatives $C_{N_{\dot{\alpha}}}$ and $C_{m_{\dot{\alpha}}}$ are formed in the same way as the two preceding examples, using the pressure coefficient, equation (30), and the relations,

$$\left.\begin{aligned}
C_{N_{\dot{\alpha}}} &= \left(\frac{\partial C_N}{\partial \frac{\dot{\alpha} l}{V}}\right)_{\dot{\alpha}\to 0} = -\frac{2}{S}\int_0^l R(x)dx \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\frac{\partial C_{p_3}}{\partial \frac{\dot{\alpha} l}{V}}\right)_{\dot{\alpha}\to 0} \sin\omega\, d\omega \\
C_{m_{\dot{\alpha}}} &= \left(\frac{\partial C_m}{\partial \frac{\dot{\alpha} l}{V}}\right)_{\dot{\alpha}\to 0} = \frac{2}{Sl}\int_0^l \left[xR(x) + \tan\theta\, R^2(x)\right]dx \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\frac{\partial C_{p_3}}{\partial \frac{\dot{\alpha} l}{V}}\right)_{\dot{\alpha}\to 0} \sin\omega\, d\omega
\end{aligned}\right\} \tag{49}$$

It should be noted that the stability derivatives are evaluated when the angle of attack is zero. This occurs when $t = 0$, so that, after taking the required time derivative in equation (30), the remaining terms multiplied by t may be eliminated. The results are,

$$\left.\begin{aligned}
C_{N_{\dot{\alpha}}} &= \frac{2}{3}\Gamma_1\left(\left(\frac{2M^2}{\Delta+\beta^2}\right)\left\{\left[1+\frac{\tau^2(1-\Delta)}{1+\Delta+2\tau^2}\right]\left[\frac{3(1-\Delta)-2(1-\beta^2\tau^2)}{4(1-\beta^2\tau^2)-3(1-\Delta)(1-\tau^2)}\right]-\right.\right. \\
&\qquad \left.\left.\frac{(1-\Delta)}{1+\Delta+2\tau^2}\right\}+\frac{(1-\Delta)}{1+\Delta+2\tau^2}\right) \\
C_{m_{\dot{\alpha}}} &= -\frac{3}{4}(1+\tau^2)C_{N_{\dot{\alpha}}}
\end{aligned}\right\} \tag{50}$$

As $\tau \to 0$, the above results reduce to the slender-body-theory result,

$$\left.\begin{aligned}
C_{N_{\dot{\alpha}}} &= \frac{2}{3} \\
C_{m_{\dot{\alpha}}} &= -\frac{1}{2}
\end{aligned}\right\} \tag{51}$$

Stability Derivatives for a Cone - Combined First- and Second-Order Theory

It has been shown by Van Dyke (ref. 1) that a further improvement in the first-order results for pressure and normal force due to inclined flow past a cone can be realized by making a second-order correction to the axial-flow potential. This is called in reference 1 a "hybrid theory." The same idea can be incorporated in the other two cases considered herein, since in all three, the axial-flow potential has the same form. Moreover, as in the purely first-order case, the resulting solutions are adaptable to the calculation of the stability derivatives of bodies of general shape by use of the technique outlined in reference 7. Unlike the inclined-flow case, however, it is not assured that by making the second-order correction the solutions corresponding to uniform pitching velocity and uniform vertical acceleration are necessarily improved. In the absence of exact numerical results with which to compare the solutions, it can only be supposed that such an improvement is likely, again in view of the fact that in the formulation of the problem no approximations are made beyond those which also exist in the inclined-flow solution. There follows a brief description of the method of applying the second-order correction to the potential for the cone.

For the case of uniform axial flow past a cone, the potential χ_o, correct to second order, as obtained from reference 1 is,

$$
\left.
\begin{aligned}
\frac{\chi_o}{Vx} = D\left[\sqrt{1 - \left(\frac{\beta r}{x}\right)^2} - \cosh^{-1}\frac{x}{\beta r}\right] + \\
E^2M^2\left\{\left(\cosh^{-1}\frac{x}{\beta r}\right)^2 - (N + 1)\sqrt{1 - \left(\frac{\beta r}{x}\right)^2}\cosh^{-1}\frac{x}{\beta r} - \right. \\
\left.\frac{E}{4}\left(\frac{x}{r}\right)^2\left[1 - \left(\frac{\beta r}{x}\right)^2\right]^{3/2}\right\}
\end{aligned}
\right\} \quad (52)
$$

where,

$$D = \frac{(1 + P - Q)\tau^2}{\sqrt{1 - \beta^2\tau^2} + \tau^2\cosh^{-1}\frac{1}{\beta\tau}}$$

$$P = \left(\frac{M}{\Delta + \beta^2}\right)^2\left[\Delta^2 - \frac{(N - 1)\beta^2\tau^2\Delta}{(1 - \beta^2\tau^2)} - \frac{(N + 1)\beta^4\tau^4}{(1 - \beta^2\tau^2)} - \frac{3}{4}\frac{\beta^6\tau^4}{(\beta^2 + \Delta)(1 - \beta^2\tau^2)}\right]$$

$$Q = \left(\frac{M}{\Delta + \beta^2}\right)^2\left[-2\beta^2\Delta + \frac{(N + 1)\beta^4\tau^2}{(1 - \beta^2\tau^2)} + \frac{(N - 1)\beta^4\tau^2\Delta}{(1 - \beta^2\tau^2)} + \frac{1}{4}\frac{\beta^6(2 + \beta^2\tau^2)}{(\beta^2 + \Delta)(1 - \beta^2\tau^2)}\right]$$

$$E = \frac{\tau^2}{\sqrt{1 - \beta^2\tau^2} + \tau^2\operatorname{sech}^{-1}\beta\tau}$$

$$N = \frac{(\gamma + 1)M^2}{2\beta^2}$$

This result applies to the uniform axial-flow potential for each of the three motions. The derivation of pressure coefficient and stability derivatives then proceeds exactly as before, with the exception that equation (52) is used in place of equation (35). The results for the stability derivatives are,

$$\left.\begin{aligned}
C_{N_\alpha} &= 2\Gamma_2\left(\frac{\mu\beta^2}{\Delta + \beta^2}\right)\left(\frac{1 + \tau^2}{1 + 2\tau^2 + \Delta}\right) \\
C_{N_q} &= \frac{2}{3}\Gamma_2\left\{1 + 2\left(\frac{\mu\beta^2}{\Delta + \beta^2}\right)(1 + \tau^2)\left[\frac{3(1-\Delta) - 2(1-\beta^2\tau^2)}{4(1-\beta^2\tau^2) - 3(1-\Delta)(1-\tau^2)}\right]\right\} \\
C_{N_{\dot{\alpha}}} &= \frac{2}{3}\Gamma_2\left(\frac{2M^2}{\beta^2}\left(\frac{\mu\beta^2}{\Delta+\beta^2}\right)\left\{\left[1+\frac{\tau^2(1-\Delta)}{1+\Delta+2\tau^2}\right]\left[\frac{3(1-\Delta) - 2(1-\beta^2\tau^2)}{4(1-\beta^2\tau^2) - 3(1-\Delta)(1-\tau^2)}\right] - \right.\right. \\
&\qquad \left.\left.\frac{(1-\Delta)}{1+\Delta+2\tau^2}\right\} + \frac{(1-\Delta)}{1+\Delta+2\tau^2}\right) \\
C_{m_\alpha} &= -\frac{2}{3}(1 + \tau^2)C_{N_\alpha} \\
C_{m_q} &= -\frac{3}{4}(1 + \tau^2)C_{N_q} \\
C_{m_{\dot{\alpha}}} &= -\frac{3}{4}(1 + \tau^2)C_{N_{\dot{\alpha}}}
\end{aligned}\right\} \tag{53}$$

where

$$\Gamma_2 = \left\{1 + 0.2\left[1 - (1+\tau^2)\left(\frac{\mu\beta^2}{\Delta+\beta^2}\right)^2\right]\right\}^{2.5}$$

and

$$\mu = \left(1 + \frac{\Delta}{\beta^2}Q + P\right)$$

Note that these results are derivable from the purely first-order solutions (eqs. (40), (45), and (50)) if the factor $\beta^2/(\Delta+\beta^2)$ is multiplied by μ wherever it appears in the first-order solutions. Therefore, since μ goes to 1 as $\tau \rightarrow 0$, these results also reduce to the slender-body-theory results (eqs. (41), (46), and (51)) as $\tau \rightarrow 0$. Note also in equations (53) that the pitching-moment coefficients are simple multiples of their respective normal-force coefficients. For a given thickness ratio, the centers of pressure are therefore invariant with Mach number.

Transfer of Axes

The results for C_{N_q}, C_{m_α}, C_{m_q}, and $C_{m_{\dot{\alpha}}}$ presented in the preceding sections are applicable only to the case of a body whose center of gravity is at the nose. These results may be used, however, to calculate the stability derivatives for any other center-of-gravity position, by means of the transfer relations given below.

$$\left.\begin{aligned}
C_{N_q} &= C_{N_{q_o}} - \frac{x_o}{l} C_{N_\alpha} \\
C_{m_\alpha} &= C_{m_{\alpha_o}} + \frac{x_o}{l} C_{N_\alpha} \\
C_{m_q} &= C_{m_{q_o}} + \frac{x_o}{l} C_{N_{q_o}} - \frac{x_o}{l} C_{m_{\alpha_o}} - \left(\frac{x_o}{l}\right)^2 C_{N_\alpha} \\
C_{m_{\dot{\alpha}}} &= C_{m_{\dot{\alpha}_o}} + \frac{x_o}{l} C_{N_{\dot{\alpha}}}
\end{aligned}\right\} \tag{54}$$

Here, x_o is the new center of gravity position, measured positive rearward from the nose, and the subscripted terms are the stability derivatives as calculated for $x_o = 0$.

Newtonian Impact Theory

Results cannot be obtained by the potential theories used in preceding sections above a Mach number for which the Mach cone from the body apex coincides with the body surface. For bodies of moderate thickness ratio, this condition limits the applicability of the theories to the Mach number range below about 3 or 4. For Mach numbers very much higher than this limit, it may be expected that the underlying assumptions of the Newtonian impact theory become increasingly valid. Therefore, in order to provide some information on the nature of the aerodynamic forces, moments, and stability derivatives at very high Mach numbers, the Newtonian theory is adapted below to the derivation of these quantities for arbitrary bodies of revolution.

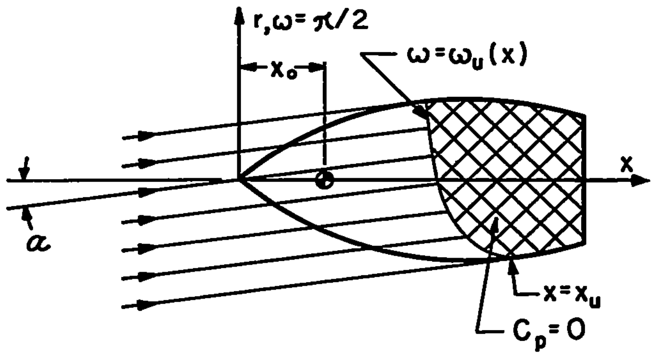
Pressure, force, and moment coefficients.- The assumption basic to impact theory is that the flow, upon striking the body, loses entirely its component of momentum normal to the surface and continues along the surface with its tangential component of momentum unchanged. The loss of momentum normal to the body surface yields a pressure force that is simply (ref. 9),

$$C_p = \frac{p - p_o}{q_o} = \frac{2V_N^2}{V^2} \tag{55}$$

where V_N is the component of velocity normal to the body. It is to be noted that only those portions of the body that "see the flow," that is, are under direct impact from the stream, will experience a pressure force. The remainder of the body is generally assumed to be at the pressure of the free stream, so that on these portions, the pressure coefficient is zero. Therefore, when integrating the local pressures over the body surface to obtain total forces and moments, the integration proceeds only over the portion of the body receiving compression flow.[2] The formulas for normal force, axial force, and pitching-moment coefficient then are,

$$\left.\begin{aligned}
C_N &= -\frac{2}{S}\int_0^{x_u} R(x)dx \int_{-\frac{\pi}{2}}^{\omega_u(x)} C_p \sin \omega \, d\omega \\
C_X &= \frac{2}{S}\int_0^{x_u} R(x)\tan \theta \, dx \int_{-\frac{\pi}{2}}^{\omega_u(x)} C_p d\omega \\
C_m &= \frac{2}{Sl}\int_0^{x_u} [(x - x_o)R(x) + \tan \theta \, R^2(x)]dx \int_{-\frac{\pi}{2}}^{\omega_u(x)} C_p \sin \omega \, d\omega
\end{aligned}\right\} \tag{56}$$

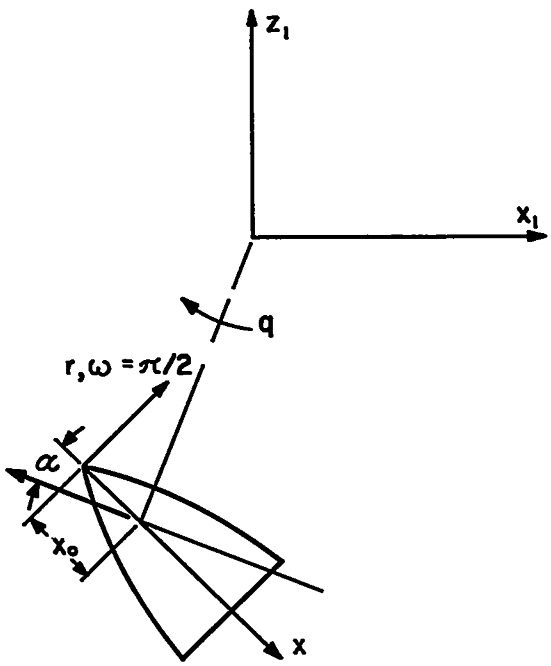
[2]Actually, in addition to the impact forces, there should be considered the centrifugal forces which arise from the fact that the flow particles follow curved paths along the body after impact (ref. 9). Some gross estimates of their magnitude have been made which indicate that they are negligibly small for the small changes in α, $\dot{\alpha}$, and q that are of primary interest here. A more precise estimation of their magnitude is a matter of some difficulty, and in view of the approximate nature of the entire theory, the effort required in this direction does not appear to be warranted. The centrifugal forces are therefore neglected in the subsequent analysis.



Sketch (d)

where the quantities $\omega_u(x)$, x_u, and x_o are illustrated in sketch (d). Provision has been made in equation (56) for an arbitrary axis of rotation x_o for reasons to be made clear in the next section.

Combined angle of attack and pitching velocity.- Since the pressure coefficient (eq. (55)) is proportional to the square of the normal component of velocity, there arises the possibility that the forces and moments due to a motion involving both pitching and angle of attack may not be treated separately, as in the case of the usual linear analysis. Therefore, we begin by investigating the forces due to a combined motion. Further, for the same reason, it is not immediately evident that the transfer relations given previously (eq. (54)) remain valid, so that rather than choosing a particular axis of rotation, we shall consider the body to be pitching about an arbitrary axis, x_o. Referred to a fixed system of coordinates, the situation is as illustrated in sketch (e). In the body system of coordinates, the components of stream velocity in the axial, radial, and azimuthal directions are,



Sketch (e)

$$\left.\begin{aligned}
u &= V - qr \sin \omega \\
v &= V \tan \alpha \sin \omega + q(x - x_o)\sin \omega \\
w &= V \tan \alpha \cos \omega + q(x - x_o)\cos \omega
\end{aligned}\right\} \qquad (57)$$

The components u and v are in turn resolved into the component V_N normal to the body surface by the relation,

$$V_N = u \sin \theta - v \cos \theta \tag{58}$$

so that,

$$V_N = V(\sin \theta - \tan \alpha \sin \omega \cos \theta) - q \sin \omega[(x - x_o)\cos \theta + r \sin \theta] \tag{59}$$

Forming the pressure coefficient according to equation (55), substituting into equation (56), and integrating once, we get,

$$\left.\begin{aligned}
C_N &= -\frac{2}{S}\int_0^{x_u} R(x)\left[-A(x)\cos \omega_u + B(x)\left(\frac{\omega_u}{2} - \frac{1}{4}\sin 2\omega_u + \frac{\pi}{4}\right) - \right.\\
&\qquad \left.\frac{2}{3}G(x)\cos \omega_u(\sin^2\omega_u + 2)\right]dx\\
C_X &= \frac{2}{S}\int_0^{x_u} R(x)\tan \theta\left[A(x)\left(\omega_u + \frac{\pi}{2}\right) - B(x)\cos \omega_u + \right.\\
&\qquad \left.2G(x)\left(\frac{\omega_u}{2} - \frac{1}{4}\sin 2\omega_u + \frac{\pi}{4}\right)\right]dx\\
C_m &= \frac{2}{Sl}\int_0^{x_u} [(x - x_o)R(x) + \tan \theta\, R^2(x)]\left[-A(x)\cos \omega_u + \right.\\
&\qquad \left.B(x)\left(\frac{\omega_u}{2} - \frac{1}{4}\sin 2\omega_u + \frac{\pi}{4}\right) - \frac{2}{3}G(x)\cos \omega_u(\sin^2\omega_u + 2)\right]dx
\end{aligned}\right\} \tag{60}$$

where

$$A(x) = 2\sin^2\theta$$

$$B(x) = -4\frac{ql}{V}\sin\theta\left[\left(\frac{x - x_o}{l}\right)\cos\theta + \frac{R}{l}\sin\theta\right] - 2\sin 2\theta\tan\alpha$$

$$G(x) = \left\{\left(\frac{ql}{V}\right)\left[\frac{R}{l}\sin\theta + \frac{(x - x_o)}{l}\cos\theta\right] + \tan\alpha\cos\theta\right\}^2$$

When the entire body experiences compression flow, the limit of integration x_u may be replaced by l, and ω_u by $\frac{\pi}{2}$, whereupon equations (60) simplify to,

$$\left.\begin{aligned}
C_N &= -\frac{\pi}{S}\int_0^l R(x)B(x)dx \\
C_X &= \frac{2\pi}{S}\int_0^l R(x)\tan\theta[A(x) + G(x)]dx \\
C_m &= \frac{\pi}{Sl}\int_0^l [(x - x_o)R(x) + \tan\theta\, R^2(x)]B(x)dx
\end{aligned}\right\} \tag{61}$$

The latter equations apply so long as the inequality given below is satisfied over the entire body:

$$\tan^{-1}\left[\frac{\tan\alpha + \frac{q}{V}(x - x_o)}{1 - qR/V}\right] \leq \theta(x) \tag{62}$$

For bodies with continually growing cross sections at small angles of attack (essentially, $\alpha < \theta(x)$), the inequality (62) will in fact be satisfied; attention in this case can, therefore, be confined to the simpler set of equations (61). It is to be noted in equations (61) that both the normal force and pitching moment depend only on $B(x)$. Then, since α must be small by virtue of equation (62), the small-angle approximation becomes valid, so that $B(x)$ may be written as the sum of two terms, each linear in q or α alone. Therefore, C_N and C_m are expressible in terms of stability derivatives in the usual way; that is,

$$\left.\begin{aligned} C_N &= \left(\frac{\partial C_N}{\partial \alpha}\right)_{\alpha \to 0} \alpha + \left(\frac{\partial C_N}{\partial \frac{ql}{V}}\right)_{q \to 0} \frac{ql}{V} \\ C_m &= \left(\frac{\partial C_m}{\partial \alpha}\right)_{\alpha \to 0} \alpha + \left(\frac{\partial C_m}{\partial \frac{ql}{V}}\right)_{q \to 0} \frac{ql}{V} \end{aligned}\right\} \tag{63}$$

Matching terms in equations (63) and (61), we find,

$$\left.\begin{aligned} \left(\frac{\partial C_N}{\partial \alpha}\right)_{\alpha \to 0} &= C_{N_\alpha} = \frac{2\pi}{S}\int_0^l R(x)\sin 2\theta\, dx \\ \left(\frac{\partial C_N}{\partial \frac{ql}{V}}\right)_{q \to 0} &= C_{N_q} = \frac{2\pi}{S}\int_0^l R(x)\left[\frac{x}{l}\sin 2\theta + 2\,\frac{R(x)}{l}\sin^2\theta\right]dx - \frac{x_0}{l}\,C_{N_\alpha} \\ \left(\frac{\partial C_m}{\partial \alpha}\right)_{\alpha \to 0} &= C_{m_\alpha} = -\,\frac{2\pi}{Sl}\int_0^l [xR(x) + \tan\theta\, R^2(x)]\sin 2\theta\, dx + \frac{x_0}{l}\,C_{N_\alpha} \\ \left(\frac{\partial C_m}{\partial \frac{ql}{V}}\right)_{q \to 0} &= C_{m_q} = -\,\frac{2\pi}{Sl}\int_0^l [xR(x) + \tan\theta\, R^2(x)]\left[\frac{x}{l}\sin 2\theta + \right. \\ &\qquad \left. 2\,\frac{R(x)}{l}\sin^2\theta\right]dx + \frac{x_0}{l}\,C_{N_{q_0}} - \frac{x_0}{l}\,C_{m_{\alpha_0}} - \left(\frac{x_0}{l}\right)^2 C_{N_\alpha} \end{aligned}\right\} \tag{64}$$

Thus, comparing terms with the transfer relations, equations (54), it can be seen that under the same conditions for which equation (63) is valid, the transfer relations are also valid, for the integrals in the last three of equations (64) correspond to the stability derivatives $C_{N_{q_o}}$, $C_{m_{\alpha_o}}$, and $C_{m_{q_o}}$, as may be verified by letting x_o be zero in equation (61).

Stability derivatives for large values of α and q.- For larger values of α and ql/V, for which the inequality (62) fails, it is necessary to return to equations (60). In this case, it is still possible to write a set of equations analogous to equations (63), although the definitions of the stability derivatives must be revised somewhat. Usually, in a stability analysis, one is interested in the small deviations from some equilibrium condition. This may be represented in equations (60) by letting α and q be,

$$\left.\begin{aligned} \alpha &= \alpha_T + \Delta\alpha \\ q &= q_T + \Delta q \end{aligned}\right\} \tag{65}$$

where α_T and q_T are the angle of attack and pitching velocity corresponding to the equilibrium (trim) condition, and $\Delta\alpha$ and Δq are small deviations in these quantities. Correspondingly, the normal-force and pitching-moment coefficients may be written,

$$\left.\begin{aligned} C_N &= C_N(\alpha_T,\ q_T) + \Delta C_N \\ C_m &= C_m(\alpha_T,\ q_T) + \Delta C_m \end{aligned}\right\} \tag{66}$$

where $C_N(\alpha_T,\ q_T)$ and $C_m(\alpha_T,\ q_T)$ are equations (60) with α and q replaced by α_T and q_T. Then, since $\Delta\alpha$ and Δq are small, use of the small-angle approximation is again permissible. Hence, retaining only first-order terms in $\Delta\alpha$ and Δq, we may write the changes ΔC_N and ΔC_m as,

$$\left.\begin{aligned} \Delta C_N &= \left(\frac{\partial C_N}{\partial \Delta\alpha}\right)_{\Delta\alpha \to o} \Delta\alpha + \left(\frac{\partial C_N}{\partial \frac{\Delta q l}{V}}\right)_{\Delta q \to o} \frac{\Delta q l}{V} \\ \Delta C_m &= \left(\frac{\partial C_m}{\partial \Delta\alpha}\right)_{\Delta\alpha \to o} \Delta\alpha + \left(\frac{\partial C_m}{\partial \frac{\Delta q l}{V}}\right)_{\Delta q \to o} \frac{\Delta q l}{V} \end{aligned}\right\} \tag{67}$$

The quantities in parentheses are the stability derivatives, corresponding to the analogous terms in equations (63). Now, however, they are to be evaluated near the equilibrium condition, and their values will depend on the angle of attack α_T and pitching velocity q_T corresponding to that condition; that is,

$$\left.\begin{aligned}
C_{N_\alpha} &= \left(\frac{\partial C_N}{\partial \Delta\alpha}\right)_{\Delta\alpha \to 0} = -\frac{2}{S}\int_0^{x_u} R(x)H(x,\ \alpha_T,\ q_T)dx \\
C_{N_q} &= \left(\frac{\partial C_N}{\partial \frac{\Delta q l}{V}}\right)_{\Delta q \to 0} = -\frac{2}{S}\int_0^{x_u} R(x)J(x,\ \alpha_T,\ q_T)dx \\
C_{m_\alpha} &= \left(\frac{\partial C_m}{\partial \Delta\alpha}\right)_{\Delta\alpha \to 0} = \frac{2}{Sl}\int_0^{x_u} [(x - x_o)R(x) + \tan\theta\ R^2(x)]H(x,\ \alpha_T,\ q_T)dx \\
C_{m_q} &= \left(\frac{\partial C_m}{\partial \frac{\Delta q l}{V}}\right)_{\Delta q \to 0} = \frac{2}{Sl}\int_0^{x_u} [(x - x_o)R(x) + \tan\theta\ R^2(x)]J(x,\ \alpha_T,\ q_T)dx
\end{aligned}\right\} \quad (68)$$

The quantities H and J are, from equations (60),

$$\left.\begin{aligned}
H &= \left(\frac{\omega_u}{2} - \frac{1}{4}\sin 2\omega_u + \frac{\pi}{4}\right)\frac{\partial B}{\partial \Delta\alpha} - \frac{2}{3}\cos\omega_u(\sin^2\omega_u + 2)\frac{\partial G}{\partial \Delta\alpha} \\
J &= \left(\frac{\omega_u}{2} - \frac{1}{4}\sin 2\omega_u + \frac{\pi}{4}\right)\frac{\partial B}{\partial\left(\frac{\Delta q l}{V}\right)} - \frac{2}{3}\cos\omega_u(\sin^2\omega_u + 2)\frac{\partial G}{\partial\left(\frac{\Delta q l}{V}\right)}
\end{aligned}\right\} \quad (69)$$

wherein

$$\frac{\partial B}{\partial \Delta\alpha} = -2 \sin 2\theta \sec^2\alpha_T$$

$$\frac{\partial G}{\partial \Delta\alpha} = \left(\frac{q_T l}{V}\right)\sec^2\alpha_T\left[\frac{R}{l} \sin 2\theta + 2\left(\frac{x - x_o}{l}\right)\cos^2\theta\right] + 2 \tan \alpha_T\sec^2\alpha_T\cos^2\theta$$

$$\frac{\partial B}{\partial\left(\frac{\Delta q l}{V}\right)} = -2\left[2 \frac{R}{l} \sin^2\theta + \left(\frac{x - x_o}{l}\right)\sin 2\theta\right]$$

$$\frac{\partial G}{\partial\left(\frac{\Delta q l}{V}\right)} = 2\left(\frac{q_T l}{V}\right)\left[\frac{R}{l} \sin \theta + \left(\frac{x - x_o}{l}\right)\cos \theta\right]^2 + \tan \alpha_T\left[\frac{R}{l} \sin 2\theta + 2\left(\frac{x - x_o}{l}\right)\cos^2\theta\right]$$

A few remarks may be in order regarding the use of equations (68). First, it can easily be verified that the transfer relations, equations (54), are still valid, so that for a given trim condition, it is only necessary to compute equations (68) for a single convenient axis position. Also, it should be noted that equations (68) actually contain (64), as may be verified by putting $\alpha_T = q_T = 0$, $\omega_u = \frac{\pi}{2}$, and $x_u = l$ in equations (68). Second, with regard to the dependence of the stability derivatives on q_T and α_T: For any practical case it is not conceivable that the pitching velocity parameter $q_T l/V$ can ever become very large; hence, the dependence of the stability derivatives on q_T is probably not significant. On the other hand, the equilibrium angle of attack α_T can conceivably be very large; in this event, the stability derivatives as evaluated by equations (68) can differ significantly from those evaluated according to equations (64) and should be used in their place.

Uniform vertical acceleration.- The impact theory, when applied to a uniformly accelerating motion, gives zero for the force and moment proportional to the acceleration parameter, $\dot{\alpha}l/V$. This result is to be expected in view of the following considerations. It is known (ref. 10) that $C_{N_{\dot{\alpha}}}$ is closely related to the build-up in normal force that occurs following a step change in angle of attack; more precisely, it is proportional to the area enclosed by the indicial response curve and the steady-state ordinate of the indicial curve. Now, implicit in the development of impact theory is the assumption that the pressure response to the impact of each flow particle is instantaneous. As a consequence, the indicial response in normal force to a step change in angle of attack is itself a step, whence it follows that the area proportional to $C_{N_{\dot{\alpha}}}$ is zero. Likewise, $C_{m_{\dot{\alpha}}}$ is proportional to the area between the indicial pitching-moment variation and its steady-state ordinate. Again, by impact theory this variation is a step, so that, for the same reason, $C_{m_{\dot{\alpha}}}$ is zero.

Stability derivatives for a cone.- Finally, in order to complete the set of stability derivatives obtained earlier by the potential theories, the stability derivatives for a cone as derived from impact theory are presented below. Calculations are based on equations (64) with $x_o = 0$, and apply only to the case wherein the independent variables (α, $\dot{\alpha}$, q) approach zero.

$$\left.\begin{aligned} C_{N_\alpha} &= \frac{2}{1 + \tau^2} \\ C_{m_{\alpha_o}} &= -\frac{4}{3} \\ C_{N_{q_o}} &= \frac{4}{3} \\ C_{m_{q_o}} &= -(1 + \tau^2) \\ C_{N_{\dot{\alpha}}} &= C_{m_{\dot{\alpha}}} = 0 \end{aligned}\right\} \qquad (70)$$

It is of interest to note (comparing eqs. (70) with eqs. (40), (45), and (53)) that the centers of pressure for the angle-of-attack and pitching cases as derived from impact theory are identical to those derived from the potential theories.

DISCUSSION OF RESULTS

In order to indicate the nature of the results as obtained from the theoretical methods developed herein, numerical calculations have been carried out for two cones having semivertex angles of 10° and 20°. Results for the variation with Mach number of the stability derivatives C_{N_α}, $C_{N_{q_o}}$, and $C_{N_{\dot{\alpha}}}$ are shown in figures 1 to 3. The pitching-moment variations are not shown since in all cases they are simple multiples of the normal-force results (see eqs. (40), (45), (50), (53), and (70)). The curves obtained from potential theory have been terminated at the low end of the Mach number scale slightly below the Mach number for which the bow wave detaches (ref. 11), and at the high end, at the Mach number for which the Mach cone lies on the body surface.

As noted before, it is believed that the results for the stability derivatives due to pitching velocity and to vertical acceleration, obtained by use of a first-order and a combination first- and second-order potential theory, are comparable in accuracy to the results for C_{N_α} obtained by the corresponding theory. Therefore, in order to estimate the Mach number range in which the results should apply, consider first the results for C_{N_α} (fig. 1). Here, a comparison is made between the various approximate results and the exact numerical result as obtained from reference 2. The curves labeled "first-order, exact C_p" and "first- and second-order, exact C_p" are those obtained from calculations based on equations (40) and (53). The curve labeled "first-order, approximate C_p" is the result, when using the first-order potential solution, of retaining only the first-order term in the expansion of the pressure relation, equation (28). It is clear from examination of the results that there is a significant improvement in accuracy, even in the first-order solution, if the pressure relation is not approximated. Even so, however, only the "hybrid" solution can be said to be applicable throughout the Mach number range for both cones. Therefore, in the subsequent results, figures 2 and 3, it is to be assumed that only the hybrid solutions are representative of the exact variations for all Mach numbers within the limits, the Mach number for bow-wave detachment, and the Mach number corresponding to $\beta\tau = 1$. Also shown in figure 1 are the results for C_{N_α} as obtained from the Newtonian impact theory (eqs. (70)). These results are useful as a guide for estimating the amount by which the hybrid theory tends to overestimate the magnitude of the normal-force coefficient at the higher Mach numbers. A fairing of the hybrid-theory result into the impact-theory result is easily accomplished and may serve partially to compensate for this tendency.

Consider next the results for C_{N_q} (fig. 2). It is noted that the curves for the various approximations are in the same relation to one another as were the curves for the corresponding approximations to C_{N_α}. Also, as in the previous case, the hybrid-theory result approaches the result obtained from impact theory in a manner which permits a judicious fairing of the two.

Finally, consider the result for $C_{N_{\dot{\alpha}}}$ (fig. 3). The interesting point here is that, in contrast to the case of a wing, $C_{N_{\dot{\alpha}}}$ as obtained from potential theory is positive throughout the Mach number range. This fact implies that at least for axes ahead of the center of loading due to $\dot{\alpha}$, the damping moment $C_{m_q} + C_{m_{\dot{\alpha}}}$ cannot be destabilizing, since the destabilizing contribution can only arise from a negative value of $C_{N_{\dot{\alpha}}}$. Actually, it can be shown that $C_{m_q} + C_{m_{\dot{\alpha}}}$ is not a destabilizing damping moment for any position of the axis by operating with the transfer relations (eq. (54)) as was done in reference 10. It is also worthy of note in figure 3 that $C_{N_{\dot{\alpha}}}$ vanishes when the Mach cone lies on the body surface. This result is consistent with that obtained from impact theory, and, as mentioned previously, implies that at this and greater Mach numbers the indicial normal-force response to a step change in angle of attack is itself a step.

Ames Aeronautical Laboratory
National Advisory Committee for Aeronautics
Moffett Field, Calif., May 21, 1956

APPENDIX A

DERIVATION OF PRESSURE COEFFICIENT

We derive here the pressure relationships used in the potential-theory analysis for each of the three motions considered herein.

Consider a body which moves past an x_1, y_1, z_1 coordinate system fixed with respect to still air. Then from Bernoulli's equation for unsteady irrotational flow, one has (ref. 12)

$$\varphi_{t_1} + \frac{1}{2}\left(\varphi_{x_1}{}^2 + \varphi_{y_1}{}^2 + \varphi_{z_1}{}^2\right) + \frac{\gamma}{\gamma - 1}\frac{p}{\rho} = \frac{\gamma}{\gamma - 1}\frac{p_o}{\rho_o} \tag{A1}$$

Since the flow is isentropic,

$$\frac{p}{\rho^\gamma} = \text{constant}$$

so that,

$$\frac{p}{\rho} = \frac{p_o^{\frac{1}{\gamma}}}{\rho_o} p^{\frac{\gamma-1}{\gamma}} \tag{A2}$$

Substituting equation (A2) in (A1) and solving for p,

$$p^{\frac{\gamma-1}{\gamma}} = p_o^{\frac{\gamma-1}{\gamma}}\left\{1 - \frac{\gamma - 1}{\gamma}\frac{\rho_o}{p_o}\left[\varphi_{t_1} + \frac{1}{2}\left(\varphi_{x_1}{}^2 + \varphi_{y_1}{}^2 + \varphi_{z_1}{}^2\right)\right]\right\} \tag{A3}$$

But $\dfrac{\gamma p_o}{\rho_o} = a_o{}^2$, where a_o is the speed of sound in still air. Then

$$p = p_o\left\{1 - \frac{\gamma - 1}{a_o{}^2}\left[\varphi_{t_1} + \frac{1}{2}\left(\varphi_{x_1}{}^2 + \varphi_{y_1}{}^2 + \varphi_{z_1}{}^2\right)\right]\right\}^{\frac{\gamma}{\gamma-1}}$$

and

$$\frac{p - p_o}{p_o} = \left\{1 - \frac{\gamma - 1}{a_o{}^2}\left[\varphi_{t_1} + \frac{1}{2}\left(\varphi_{x_1}{}^2 + \varphi_{y_1}{}^2 + \varphi_{z_1}{}^2\right)\right]\right\}^{\frac{\gamma}{\gamma-1}} - 1 \tag{A4}$$

A reference velocity V is now introduced; we define it to be the velocity of the body in its axial direction, since this will be a constant for all three motions. Then setting

$$p_o = \frac{2}{\gamma M^2} q_o \left(M^2 = \frac{V^2}{{a_o}^2}, \quad q_o = \frac{1}{2} \rho_o V^2\right)$$

we have,

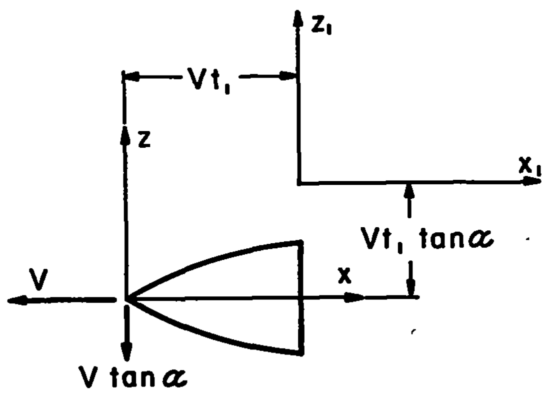
$$\frac{p - p_o}{q_o} = C_p = \frac{2}{\gamma M^2} \left\{ \left[1 - \frac{\gamma - 1}{2} M^2 \left(\frac{{\varphi_{x_1}}^2 + {\varphi_{y_1}}^2 + {\varphi_{z_1}}^2 + 2\varphi_{t_1}}{V^2}\right)\right]^{\frac{\gamma}{\gamma-1}} - 1\right\} \tag{A5}$$

This equation refers to axes fixed with respect to still air; we next consider the transformations to body axes for each of the three body motions.

Sinking With Uniform Vertical Velocity

As can be seen on sketch (f), the fixed and moving axis systems are related by,

$$\left.\begin{aligned} x &= x_1 + Vt_1 \\ y &= y_1 \\ z &= z_1 + Vt_1 \tan\alpha \\ t &= t_1 \end{aligned}\right\} \tag{A6}$$



Sketch (f)

Then taking derivatives according to,

$$\begin{aligned} \varphi_{x_1} &= \varphi_x \\ \varphi_{y_1} &= \varphi_y \\ \varphi_{z_1} &= \varphi_z \\ \varphi_{t_1} &= \varphi_x \frac{\partial x}{\partial t_1} + \varphi_z \frac{\partial z}{\partial t_1} + \varphi_t \end{aligned}$$

equation (A5) becomes, in the body x, y, z coordinate system,

$$C_p = \frac{2}{\gamma M^2}\left\{\left[1-\frac{\gamma-1}{2}M^2\left(\frac{2\varphi_x}{V}+\frac{2\varphi_z}{V}\tan\alpha+\frac{2\varphi_t}{V^2}+\frac{\varphi_x{}^2+\varphi_y{}^2+\varphi_z{}^2}{V^2}\right)\right]^{\frac{\gamma}{\gamma-1}}-1\right\} \tag{A7}$$

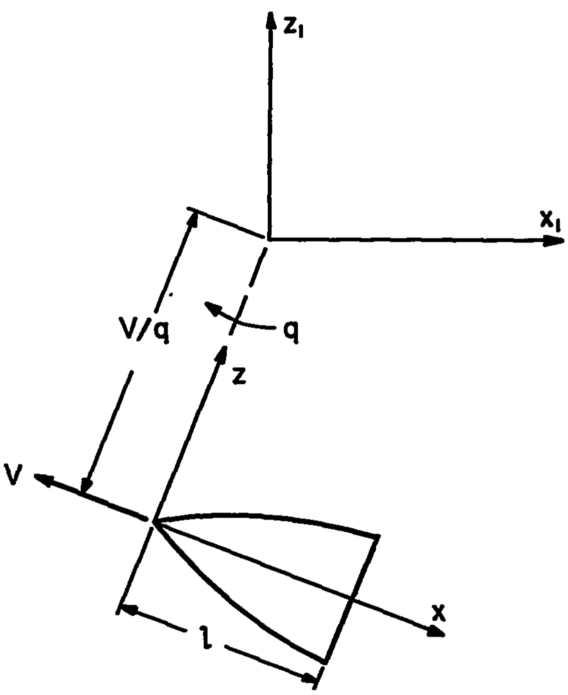
Finally, converting to cylindrical coordinates by the transformations,

$$\left.\begin{aligned} x &= x \\ y &= r\cos\omega \\ z &= r\sin\omega \end{aligned}\right\} \tag{A8}$$

and letting φ_t be zero for steady flow, we have for C_p in the moving axis system,

$$C_{p_1} = \frac{2}{\gamma M^2}\left(\left\{1-\frac{\gamma-1}{2}M^2\left[\frac{2\varphi_x}{V}+2\tan\alpha\left(\frac{\varphi_\omega}{rV}\cos\omega+\frac{\varphi_r}{V}\sin\omega\right)+\frac{\varphi_x{}^2+\varphi_r{}^2+(\varphi_\omega/r)^2}{V^2}\right]\right\}^{\frac{\gamma}{\gamma-1}}-1\right) \tag{A9}$$

Pitching With Uniform Angular Velocity



Sketch (g)

The relation between the fixed and moving axes in this case is illustrated in sketch (g). The transformation equations are,

$$\left.\begin{aligned} x &= x_1\cos qt_1 - z_1\sin qt_1 \\ y &= y_1 \\ z &= \frac{V}{q} + x_1\sin qt_1 + z_1\cos qt_1 \\ t &= t_1 \end{aligned}\right\} \tag{A10}$$

Equation (A5) becomes

$$C_p = \frac{2}{\gamma M^2}\left\{\left[1 - \frac{\gamma - 1}{2} M^2\left(\frac{2\varphi_x}{V} - \frac{2\varphi_x}{V^2} qz + \frac{2\varphi_z}{V^2} qx + \frac{2\varphi_t}{V^2} + \frac{\varphi_x^2 + \varphi_y^2 + \varphi_z^2}{V^2}\right)\right]^{\frac{\gamma}{\gamma-1}} - 1\right\} \tag{A11}$$

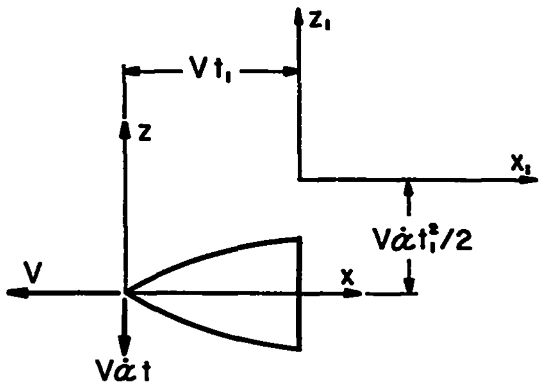
Changing to cylindrical coordinates by equation (A8), and letting φ_t be zero for steady flow,

$$C_{p_2} = \frac{2}{\gamma M^2}\left(\left\{1 - \frac{\gamma - 1}{2} M^2\left[\frac{2\varphi_x}{V} - \frac{2\varphi_x}{V^2} qr \sin\omega + \frac{2qx}{V}\left(\frac{\varphi_r}{V}\sin\omega + \frac{\varphi_\omega}{rV}\cos\omega\right) + \frac{\varphi_x^2 + \varphi_r^2 + (\varphi_\omega/r)^2}{V^2}\right]\right\}^{\frac{\gamma}{\gamma-1}} - 1\right) \tag{A12}$$

Sinking With Uniform Vertical Acceleration

Here, the relations between the fixed and moving axes are (sketch (h))

$$\left.\begin{aligned} x &= x_1 + Vt_1 \\ y &= y_1 \\ z &= z_1 + \frac{\dot{\alpha}Vt_1^2}{2} \\ t &= t_1 \end{aligned}\right\} \tag{A13}$$



Sketch (h)

The pressure equation (A5) then becomes,

$$C_p = \frac{2}{\gamma M^2}\left\{\left[1 - \frac{\gamma - 1}{2} M^2\left(\frac{2\varphi_x}{V} + \frac{2\varphi_t}{V^2} + \frac{2\dot{\alpha}t\varphi_z}{V} + \frac{\varphi_x{}^2 + \varphi_y{}^2 + \varphi_z{}^2}{V^2}\right)\right]^{\frac{\gamma}{\gamma-1}} - 1\right\} \tag{A14}$$

Converting to cylindrical coordinates by equation (A8),

$$C_{p_3} = \frac{2}{\gamma M^2}\left(\left\{1 - \frac{\gamma - 1}{2} M^2\left[\frac{2\varphi_x}{V} + \frac{2\varphi_t}{V^2} + 2\dot{\alpha}t\left(\frac{\varphi_\omega}{rV} \cos \omega + \frac{\varphi_r}{V} \sin \omega\right) + \frac{\varphi_x{}^2 + \varphi_r{}^2 + (\varphi_\omega/r)^2}{V^2}\right]\right\}^{\frac{\gamma}{\gamma-1}} - 1\right) \tag{A15}$$

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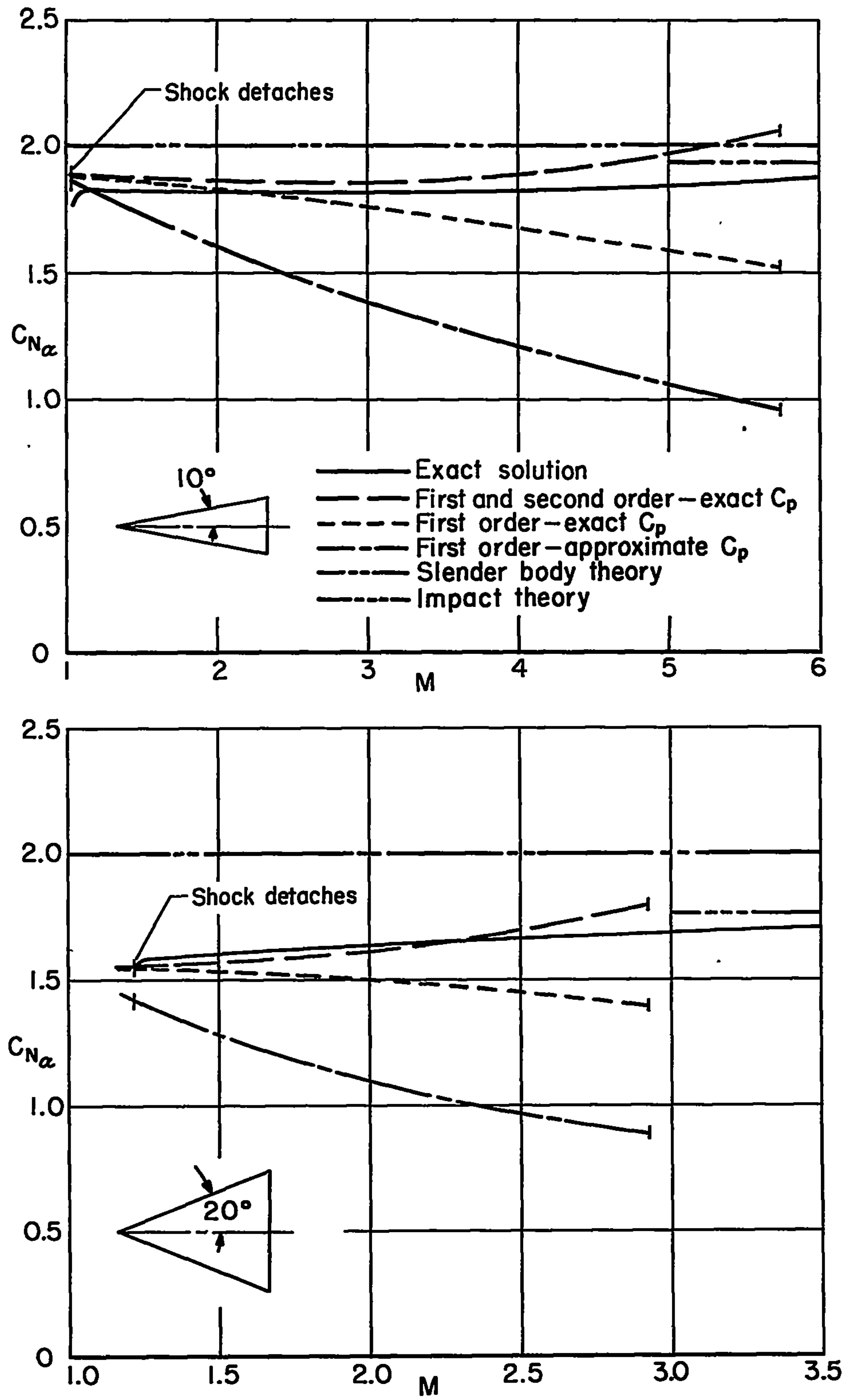

Figure 1.– Variation with Mach number of the stability derivative C_{N_α}.

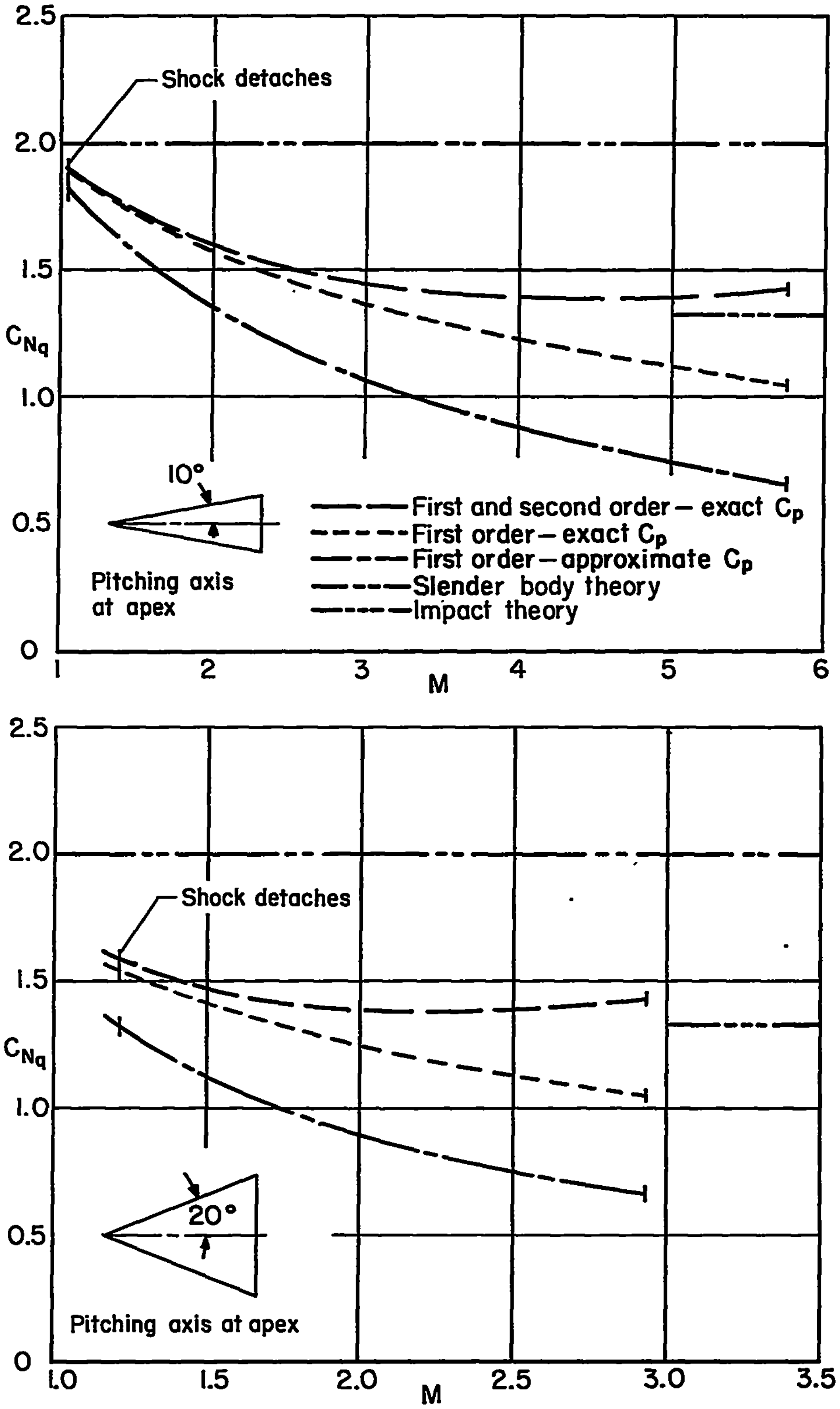

Figure 2.—Variation with Mach number of the stability derivative C_{N_q}.

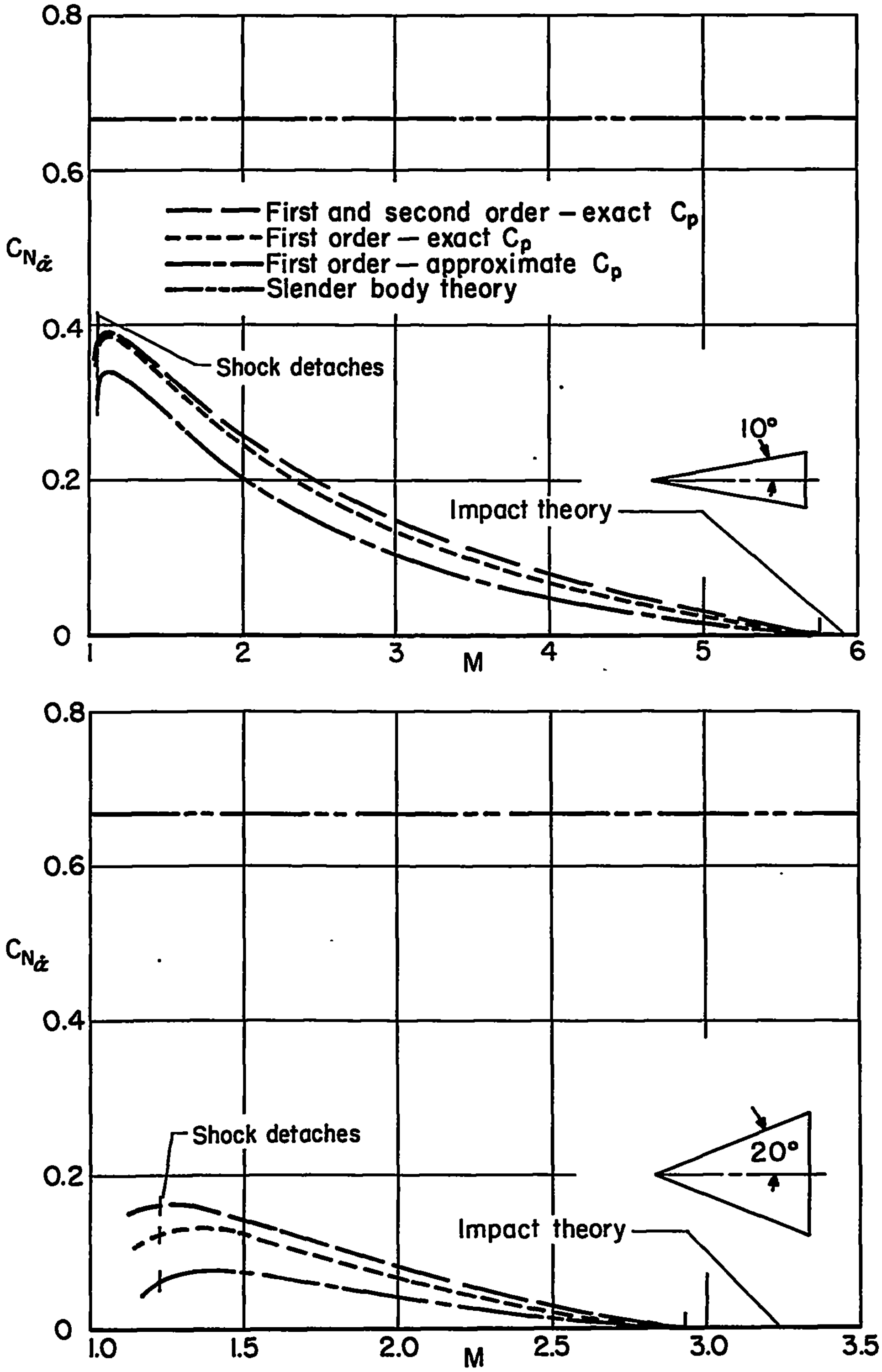

Figure 3.— Variation with Mach number of the stability derivative $C_{N_{\dot{\alpha}}}$.